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How do restrictive zoning and parental school choices impact social diversity in schools? An empirical evaluation in France[1]

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Résumé

En France, la répartition des élèves entre collèges publics se fait principalement en fonction de la carte scolaire, les élèves étant en général affectés à un unique collège public en fonction de leur lieu de résidence. Les familles qui ne sont pas satisfaites de cette affectation peuvent demander une dérogation pour un autre collège public ou scolariser leur enfant dans un collège privé. Les conséquences de ces modes d'affectation des élèves dans les établissements scolaires sur la diversité sociale de ceux-ci ne sont pas connus a priori. La composition sociale des établissements reflète en partie la ségrégation observée au niveau des quartiers, et ce d'autant plus que la carte scolaire est appliquée de manière stricte. D'un autre côté, les possibilités de contourner la carte scolaire peuvent également contribuer à polariser les compositions sociales des établissements scolaires, si ce sont principalement les catégories sociales les plus favorisées qui utilisent ce choix. Cette étude propose donc une décomposition d'un indicateur de ségrégation sociale (l'indice d'entropie) en plusieurs composantes correspondant respectivement à la ségrégation résidentielle (selon les secteurs de collège) et de celles de l'évitement vers un autre collège public, ou un collège privé. Nous utilisons des données exhaustives géolocalisées sur les élèves scolarisés au collège pour trois villes pour lesquelles nous disposons des contours précis de la carte scolaire afin de

[1]Aude Cadoret, Pascale Guillois and Olivier Sauvaitre provided us with the geolocalized data matched to the areas of the school map. The DASCO of the city of Paris provided us with the shape of these areas as they were in 2015. The authors are also grateful to Cédric Afsa, Alice Desrosier, Gabrielle Fack, Julien Grenet, Elise Lhuillery, Fabrice Murat and Jean-Christophe Vergnaud for stimulating discussions as well as seminar participants at the DEPP, LEER Workshop in Education Economics, JMA, Workshop Matching in Practice. We remain solely responsible for the content and possible errors.

quantifier les différents éléments de la ségrégation sociale au niveau des collèges. Pour les trois villes étudiées, l'évitement conduit à augmenter de 50 % à 100 % le niveau de ségrégation par rapport au niveau qui prévaudrait si les élèves étaient scolarisés dans leur collège de quartier. L'évitement vers des collèges du secteur privé explique l'essentiel de cette augmentation de la ségrégation scolaire. La composition sociale des élèves scolarisés dans un collège privé se distingue nettement de celle des élèves qui restent scolarisés dans leur collège de quartier.

Abstract

This paper provides new empirical evidence on how restrictive zoning and parental school choices impact social diversity in schools, based on French data. In France, school assignment is mainly made on a residence only based, but parents may opt out for state-subsidized private schools or, under strict conditions, another public school. Segregation at school is thus expected to mirror the residential segregation, but this may be either mitigated or inflated by opting out to another school. Using an exhaustive administrative geolocalized dataset on three French urban areas (corresponding to approximately 30,000 pupils aged 11-12), we decompose the segregation indices at the school level such as we measure the relative contributions of residential segregation and of circumvention. In the areas studied here, the choice of some parents to opting out, mainly to private schools, increases social segregation in schools by as much as 50% to 100%.

Introduction

Social diversity in schools has become an important issue in the public debate. School segregation is accused of being at least partly responsible for the observed social inequalities in terms of academic achievements. The concentration of pupils from disadvantaged backgrounds in low performing schools is a much discussed subject in the extensive literature on peer effects at school. Low social diversity at school may also hamper the creation of social cohesion.

A key question in this context is the extent to how much school choice should be given to parents. In most school systems, the enrollment of pupils in schools is based on priority rules, of which the distance to school is the most prevalent. This simple mechanism thus leads to a *de facto* restriction of the social mix in schools, since urban neighborhoods are often characterized by strong residential segregation. In turn, differences in the social composition of schools may exacerbate the relegation of some neighborhoods. The most favored families are encouraged to locate to the neighborhoods where the most reputable schools are established (for those who can afford the extra cost of housing due to the proximity of these schools).

Proponents of school choice thus argue that more choice may allow equal access to high quality schools for all pupils. While well-off families may be given a choice through residential mobility, pupils from disadvantaged backgrounds usually live in socially segregated areas and attend low performing schools. School choice may also increase competition among schools and thus foster school efficiency. Several countries have recently increased the school choice level (for a survey, see for instance OECD [2010]). Charter school policies in the United States or, more recently, policies implemented in Sweden are some examples.

On the other hand, some people emphasize that school choice may intensify school segregation. In practice, parents who are the most dedicated to school choice are often from high- or middle- class backgrounds. School seats are limited, and a very attractive school will not always be able to enroll all pupils who are applying. Eventually, it may lead to stratification according to social or schooling level.

This paper proposes new empirical evidence to this debate. It tries to quantify, in the French context, the relative contributions of residential segregation and selective choices made by families on the social diversity in schools, measured at the beginning of the French *"collège"*

level (corresponding to junior high school or middle school, enrolling pupils aged 11 - 15). The French system has the specificity of being rather restrictive for families. The assignment to a public middle school depends firstly on the place of residence according to an accurate school zoning. However, a quite large share of children (which represented 32% of children entering middle school in 2007[2]) attend a school which does not correspond to their place of residence.

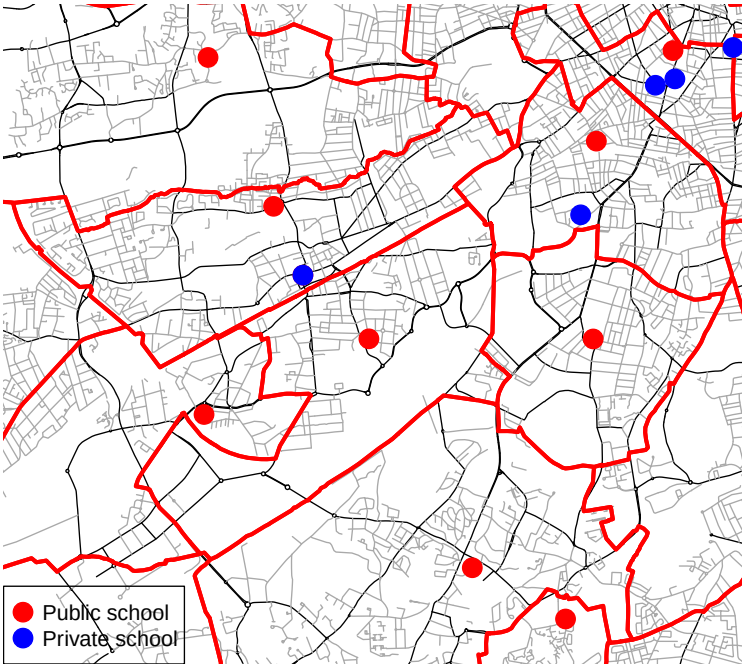
A minority of children are in a public school which is not the one assigned by the school map. This share represented 11% of children entering middle school in 2007. It includes two very different situations. First the zoning rule does not hold if the child was assigned to "special educational needs tracks" at the end of primary school. These tracks cater for children with significant learning difficulties or non-native speakers, and are offered only in certain schools (3% of children entering middle school in 2007, among which a half were not in the school corresponding to their school zone). Second, families who are not satisfied with the school corresponding to their place of residence may request a waiver to enroll for another public school. Since 2007, the waiving possibilities were enlarged and parents' motivations were given different degrees of priority. Being disabled, having special medical needs, being a grant holder are the most valued criteria. Seeking a specific curriculum (for instance, a rare second or third language such as Russian or Chinese) is among the less valued criteria. However it represents a quite big share of accepted waivers and even the majority in Paris (see Merle [2011]).

The last possibility is to opt for private education. In 2007, 21% of pupils entering middle school were enrolled in a private school (22% in 2015) and this share has remained quite stable since the 1980's. For historical reasons, French private schools have very strong ties with the public administration. The huge majority of junior high schools are linked to the state through a "contrat d'association" (92% of middle schools, which gathered 99% of pupils in 2015). They commit on recruiting teachers with the same requirements as the public sector does, and are required to teach the same hourly volumes and pedagogical content as in the public sector. Three quarters of the expenses of these schools are subsidized by the state (including the wages of teachers) or local authorities. In contrast to public schools, private schools are not subject to restrictive zoning (see Figure 1), have more autonomy in their management, are allowed to propose optional religious instruction classes, and can charge fees. Although the average fees are rather low, they are a first way through which schools can have an impact on the composition of their pupils.

A second way the composition of pupils in private schools may be influenced arises from a specific admission process. Interested parents contact the school and are very often asked to meet the head teacher. The whole process appears to be selective and may even be discriminatory. Moreover, private schools are mostly located in central neighborhoods (and boarding schools are very rare): the location of these schools can also have an impact on their recruitment, especially if they are most often located in affluent districts (see Givord et al. [2016] for an analysis on Paris and Marseilles agglomerations). As a consequence, on average, French private schools enroll pupils from more privileged background, but not only. At the national level, 20% of pupils enrolled in private schools are from disadvantaged families, for instance. This is more than twice lower than in the public schools, but not negligible.

The consequences of the coexistence of public and private educational systems on social diversity at school are unclear. Several recent contributions insist on the consequences for the social composition of public schools of the competition exercised by private ones. Givord et al. [2016] observe, for example, local polarization phenomena between private and public schools, particularly in the rather disadvantaged areas of the Paris and Marseilles agglomerations. Moreover, private schools present a large range of profiles and this diversity within the private sector contributes also to the overall segregation.

[2]Parents from the panel of French students entering sixth grade in 2007 were asked whether their child attended the school assigned by the school zoning, which allowed to estimate this share on a national scale, see Thaurel-Richard and Murat [2013].



The red lines correspond to school zoning boundaries, with each corresponding state school indicated in red and private schools in blue (a district may contain several or no private schools).

Figure 1: School zoning example in a suburban area

Decomposing the impact of residential segregation from selective choice behavior requires accurate data on both the residential address of pupils and different schools. In this paper, we use an exhaustive database of middle school pupils across three French urban areas (Paris, Bordeaux and Clermont-Ferrand) for which we know both the exact outlines of school zones and the addresses of the pupils in 2015. This makes it possible to identify for each student the school to which they would be attached if they respected the school zoning, but also to detect cases where this assigned school is circumvented. We can then compare the level of school segregation that would be observed if all the pupils in a school zone were enrolled in their public neighborhood school, to the level of school segregation actually observed in middle schools. The first indicator reflects the share of school segregation that results from urban segregation, while the gap between the two measures makes it possible to assess the extent to which families' choices to circumvent this school map aggravate or, on the contrary, lead to the resorption of a part of this residential segregation. One may then quantify the contributions of the avoidance respectively to private schooling and to another public school that the sector school. The use of decomposable segregation indices also makes it possible to identify finer dynamics. In particular, we can distinguish the impact on the social mix in avoided sectors - a priori negative - from the effect on the social mix in schools which pupils eventually join - whose sign is undetermined.

Our results confirm that the level of segregation observed at the middle school level reflects for a large part the high level of urban segregation, consistently with the fact that the French enrollment system is closely related to residential locations. However, school choice, and especially to private schools, contributes to reduce even further the social mix in middle schools. Selective choice decreases the level of social diversity at the middle school level by 50% to 100% compared to what would have been observed if only urban segregation prevailed. This result is, for the three districts studied here, essentially driven by avoidance towards private schooling. Our results allow us to go further in this decomposition. We show that this effect is linked

to local polarization. In a neighborhood, students enrolling in private schooling are often the most advantaged, and their defection further reduces the social mix in the avoided area. This effect accounts for most of the contribution of private schools. A residual effect is related to stratification among private schools. Pupils enrolling in private schools, who do not all have homogeneous social profiles even if they are on average more advantaged, are not randomly distributed among private schools.

The next section underlines the main elements of school choice systems and their potential effects on segregation among schools. The third section proposes a decomposition of social segregation measured at the middle school level, between a part linked to urban segregation among school zones, and another resulting from the circumvention of the local state school. The results are presented in the last section.

1 School choice systems and segregation

For any school system considered, the distribution of pupils between schools is related to the place of residence of their parents. Even if the choice of school is left entirely free, geographical proximity is one of the main determinants of this choice. Fack et al. [2014] have shown, in the case of the Paris region, that regarding high schools (for which school choice is relatively free within a set of possibilities), families prefer schools that are near to their home, even if they also take into account the academic level of the high school considered. School choice regulation may also lead to tighten (or more rarely relax) this geographical constraint. The relaxing of the geographical constraint would correspond, for example, to busing policies allowing pupils to attend an alternative school than their local one. A recent example is the busing policy implemented between 2000 and 2010 in *Wake County*, North Carolina (district of about 150,000 pupils), which was not aimed at ethnic integration but at the integration of students in terms of income.

In most school regulation systems, the allocation rules tend to increase the initial geographical constraint. There are two ways that this choice may be constrained: on the side of the pupils, restricting the schools to which they may apply to a given geographical area. The French example of the allocation of pupils to intermediate schools constitutes a case where the assignment to a state school is restricted to a single institution, depending on one's residential location. The geographical constraint is less strong in other systems: in the United Kingdom for example, where school catchment areas are overlapping.[3] Alternatively, the geographical constraint may stem from the method of selection of pupils by schools: even if pupils could apply freely to different schools, if geographic proximity is a criterion for selection by schools, the link between place of residence and place of schooling will be strengthened.

These various elements of geographical constraint are themselves widely attenuated by the existence of more specific policies allowing more freedom in school choice. Private schools are the most widespread example of a way in which the geographical constraint on assignment to a school may be relaxed and these may be directly funded to a large extent by the state, as it is the case for French private schools, or indirectly *via* school vouchers. Policies allowing school choice among public schools also exist. In the United States, where assignment of a pupil to a school is by default at the local district level according to the geographical residence area, charter schools as well as magnet schools have a much wider recruitment area. Owens et al. [2016] highlight the fact that the growing importance of these types of institutions in the American school system has helped to loosen the link between school segregation and residential segregation.

The policies mentioned above are sometimes restricted to specific populations, in which case the direction of the effect on segregation is relatively straightforward. This is the case for busing or certain specific voucher policies specifically designed to lessen segregation. In most cases,

[3]See OECD [2010] for a review of school choice systems in OECD countries.

however, public policy relaxes or restricts freedom of choice for all families. A central point for evaluating these policies therefore lies in who is taking up on this freedom of choice. The implications in terms of segregation will depend directly on the answers provided.

We can distinguish two potential effects of a freer school choice on school diversity. A potentially negative effect on the one hand, if for example only the most advantaged families resort to school choice, in order to join schools more advantaged than their sector one. There may also be a positive effect on social diversity if, for example, freedom of choice allows modest families, territorially constrained to reside in disadvantaged neighborhoods, to join more prestigious institutions (in the public as in the private sectors). Arguments of this type have been put forward in other reforms allowing more freedom of choice in schools, for example in Sweden in 2000: a large-scale school reform has sought to minimize the effects of residential segregation by allowing access to the most prestigious schools for all pupils, without any constraint of residence area as was the case before. Söderström and Uusitalo [2010] show that the reform has had the opposite effect of the one envisaged on social diversity, contributing rather to decrease it. It is also a potential mixing effect that was put forward during the relaxation of the school map that took place in France in 2007. However, Fack and Grenet [2012] find that the results have been mixed: the device would have mainly led to a more pronounced avoidance of the most disadvantaged sectors, but not by the pupils most directly targeted by the reform (and in particular the scholarship pupils). Musset [2012] shows that most of the studies on the impact of an increased parental choice on segregation generally conclude in an increase in segregation (for example Hsieh and Urquiola [2006] for Chile's private school voucher reform).

These different factors driving parental school choice will result in a gap between between residential segregation and school segregation that is more or less pronounced. The link between residential segregation and school segregation has already been addressed in several studies. Rangvid [2007] shows for Copenhagen that low (ethnic) residential segregation does not necessarily translate into moderate school segregation, and that the level of school segregation is mainly driven by school choice (in particular private school choice). For Germany, where primary school districts are not entirely binding, Riedel et al. [2010] have shown that school choices made by advantaged families tend to increase school segregation relative to residential segregation. In the United Kingdom (where Allen [2007] examines social segregation and segregation in terms of educational attainment of parents, and Johnston et al. [2006] ethnic segregation), residential segregation is essentially measured from local administrative divisions. Harris et al. [2007] attempt to model the catchment areas of schools to obtain a measure of residential segregation that is more comparable to that of school segregation. This question is simplified in the French case, since residential segregation can be directly measured from the institutional zoning that partitions the territory (unlike the British case).

To our knowledge, two works decompose the transition between residential segregation and school segregation between middle schools in France. Both works rely on simulation techniques applied to Paris (François [2002]]) and to Clermont-Ferrand and its surroundings (Cadoret [2017]]). The level of segregation between school zones corresponds to the segregation under the hypothesis that every pupil is in the school assigned by the school map. This defines the contribution of residential segregation to school segregation. The difference between the actual level of segregation among middle schools and the level of residential segregation is the contribution of school choices which circumvent the school map. In another scenario, the authors compute the level of segregation with pupils in their chosen school for families who chose the private sector, or in the school assigned by the school map otherwise. The difference between the latter computation and the contribution of residential segregation gives the contribution of private sector to the global segregation. Further scenarios are computed, focusing on specific social categories or specific motives for circumventing the school map.

Our work proposes a new strategy which relies on a mathematical decomposition of the segregation index rather than simulation techniques. The results are coherent with decomposition

stemming from simulations. They allow to go further in the analysis of segregation, by isolating two additionnal contributions of school choices. The first one is a "cream-skimming" effect, since pupils who circumvent the school assigned by the school map are generally from a more privileged background than pupils who don't. In most cases, it should then raise segregation. The second one is the contribution from the point of view of the schools that are joined, and whose composition is modified by these moves, which is a "reallocation effect". It could lower or raise segregation. As we mentioned before, distinguinshing both type of mechanisms is an important tool for the debate on the effect of school choice.

2 Decomposition of school segregation indices

2.1 Data and social background information

The analysis is conducted on three different urban areas, observed for year 2015: Paris, Bordeaux and Clermont-Ferrand. The data relies first on an exhaustive administrative source on middle and high school students, the *Scolarité* database, including their address geolocation. This database is combined with the accurate boundaries of school zoning. We can therefore identify, for each pupil, the public school he/she is assigned by school zoning, and the school where he/she is actually enrolled (and thus identify cases of school circumvention). We focus on middle school new entrants, usually aged 11 for those who have not previously repeated a grade.[4] We leave out of the analysis pupils who are attending "special educational needs" tracks, which are provided in a limited set of schools.[5] Among the 29,680 remaining pupils, 11% to 12% are avoiding their assigned public school in order to attend another public school (Table 1), and 20% to 30% are attending a private school. We refer to the first type as "Movers" to public schools (M^{pu}), to the second type as "Movers" to private schools (M^{pr}), and to the remaining pupils i.e. those who attend their assigned local school, as "Stayers" (S).

Table 1: Proportions of stayers and movers

		Proportion of		
	Number of pupils	M^{pr}	M^{pu}	S
Paris	18850	0.30	0.12	0.58
Bordeaux	7927	0.21	0.11	0.68
Clermont-Ferrand	2903	0.23	0.12	0.65

The socioeconomic background of pupils is known at a fine level, through the occupation of both parents, using the French classification of *professions et catégories socioprofessionnelles* (PCS). While this detailed classification is directly available (32 categories), using it may lead to high levels of segregation in a purely mechanical way (as some categories may be totally

[4]We only retain pupils who are both resident and schooled within each of the three urban areas considered, meaning that we leave aside some pupils who reside in the considered area but are schooled elsewhere, and conversely some pupils who are schooled there but reside outside. We choose to consider quite a large area around the cities of Clermont-Ferrand and Bordeaux, in order to minimize the number of pupils in such cases. However this perimeter must be restricted to dense areas where schooling alternatives actually exist. For these two areas, we then rely on the perimeter of "intercommunalities" (French *établissements publics de coopération intercommunale, EPCI)*. Those are defined by a group of communes which gather together to share the charge of managing some services, like public transportation, collecting garbage or running water. However this choice was not possible nor relevant for Paris, for which we rely on the sole city area.

[5]Such tracks may enhance segregation, but we choose not to take them into account here, since they do not correspond to typical school circumvention strategies.

absent of some schools). We therefore mainly rely on a four-category classification of parental occupations:[6] "Very advantaged" (including for example managers, teachers and professionals), "Advantaged" (including foremen, technicians), "Intermediate" (including employees) and "Disadvantaged" (manual workers or unemployed). This classification is specifically traditionally used by the French Ministry of Education to study educational issues. As an alternative we rely on a continuous index of the social position of the household, the social position index (SPI) introduced by Rocher [2016] (see Robustness analysis for more detail).

Table 2 presents the social composition of pupils in each urban area, depending on their type relative to school circumvention (M^{pr} / M^{pu} / S). Although the overall social composition differs between the three urban areas (with Paris having 50% of "Very advantaged" pupils, Bordeaux 36% and Clermont-Ferrand 29%, the pattern is generally the same regarding the relative social background of movers to the public and private sectors: for each urban area, the pupils attending private schools are more often "Very advantaged", although a few disadvantaged pupils are also concerned (more often so in Clermont-Ferrand and Bordeaux). Quite unexpectedly, the socioeconomic background of movers to another public school is slightly lower than that of the pupils who attend their local assigned school.

Table 2: Social composition in each group

Paris	Movers (private sector)	Movers (public sector)	Stayers	All
Very advantaged	0.73	0.39	0.40	0.50
Advantaged	0.07	0.08	0.08	0.08
Intermediate	0.16	0.27	0.26	0.23
Disadvantaged	0.04	0.26	0.26	0.19

Bordeaux	Movers (private sector)	Movers (public sector)	Stayers	All
Very advantaged	0.57	0.27	0.31	0.36
Advantaged	0.11	0.13	0.13	0.13
Intermediate	0.22	0.29	0.26	0.25
Disadvantaged	0.10	0.32	0.31	0.27

Clermont-Ferrand	Movers (private sector)	Movers (public sector)	Stayers	All
Very advantaged	0.41	0.24	0.26	0.29
Advantaged	0.15	0.09	0.13	0.13
Intermediate	0.28	0.24	0.24	0.25
Disadvantaged	0.16	0.43	0.37	0.33

2.2 Decomposing social segregation at school

Numerous measures have been proposed in the literature to measure segregation at school (see Frankel and Volij [2011]), as the association between schools and socioeconomic background. Here, we rely on a four-group classification, as using a binary index would entail too large a loss of information. Several segregation indices can be used in a multi-group version, among

[6] Occupation of the "primary" parent, which by default is the father. The mother's occupation is taken into account when no information on the father's occupation is available.

which the normalized entropy index H (see Reardon and Firebaugh [2002]). When considering a population of pupils that can be described by four social groups, and are enrolled in $k = 1 \dots K$ schools, the normalized entropy index is defined as:

$$\mathcal{H} = \sum_{k=1}^{K} \pi_k \frac{h(P) - h(p_k)}{h(P)} \tag{1}$$

where $P = (q_1, q_2, q_3, q_4)$ is the distribution of the four social origin types in the whole population, $p_k = (q_1^k, q_2^k, q_3^k, q_4^k)$ is the distribution of the four types in school k, $h(P) = \sum_{m=1}^{4} q_m ln(1/q_m)$ and π_k is the proportion of students in school k. This index may be seen as an "evenness" measure, when relying on usual classification of different dimensions of segregation (see Massey and Denton [1988]). It measures to which extent groups are evenly distributed among schools.

In order to measure how school choice (in particular circumvention to private school) affects school segregation, indices that verify an additive decomposition property should be favored. This property states that if we split the set of schools into two subsets, the segregation index measured in the all set of schools can be additively decomposed in a component between two sets of schools (for instance, private schools versus public schools), and two additional components that measure the level of school segregation within each of these sets respectively. The additive decomposition property states that:

$$I = I_{\text{Public vs. Private}} + \lambda_{\text{Public}} I_{\text{Public}} + \lambda_{\text{Private}} I_{\text{Private}} \tag{2}$$

where I is an additive decomposable segregation index measured using all schools, I_{Public} (respectively I_{Private}) the one using only public (respectively private) schools (and λs a weighting scheme), and $I_{\text{Public vs. Private}}$ the segregation index comparing the distribution in social characteristics of all pupils in public schools with the one for all pupils in private schools. The normalized entropy index H satisfies such a property.[7]

This decomposition is presented in Table 3. Whatever the urban area considered, the largest contribution is that of the public sector, although it is smaller for Paris. The contribution of the public-private segmentation ranges from 20% in Clermont to 33% in Paris. The contribution of the social segmentation *within* the private sector is also substantial (around 17%-20%). This gives us a first picture of the contribution of private schools to social segregation among schools.

However, comparing public and private schools as a whole provides only a partial answer regarding the contribution of school choice to segregation. Indeed, the restrictive zoning may itself generate social disparities reflecting the existing residential segregation: French urban cities are usually highly segregated by income, and as school zoning is based on residential location, it induces school social segregation. The above decomposition does not account for initial residential segregation. For instance, for historical reasons, most private schools are located in relatively well-off neighborhoods. Although these schools are not subject to the restrictive zoning, the distance criterion is important[8] as many parents prefer to choose a school located close to their home. This may lead to private schools being mainly attended by advantaged pupils, thereby driving the public-private segmentation, although this would solely reflect residential segregation. In the same way, the intra-private segmentation could reflect residential segmentation among neighborhoods where private schools are implemented. Finally, private schools are not the sole way to exert school choice. According to qualitative studies, the circumvention of the assigned public school towards an alternative public school may be quite high in some urban areas. A measure of the contribution of school choice on school segregation should take this point into account.

[7] As an alternative when using the continuous social position measure, we rely on the variance index, which is also decomposable in such a way (see Robustness analysis).

[8] Private schools are usually not boarding schools in France.

Table 3: Aggregate public sector- private sector decomposition of segregation

Paris	Entropy share	% of school segregation
Total school segregation	0.149	100%
$I_{\text{Public vs. Private}}$	0.050	33.3%
$\lambda_{\text{Public}} I_{\text{Public}}$	0.069	46.2%
$\lambda_{\text{Private}} I_{\text{Private}}$	0.031	20.5%
Bordeaux	**Entropy share**	**% of school segregation**
Total school segregation	0.099	100%
$I_{\text{Public vs. Private}}$	0.025	24.7%
$\lambda_{\text{Public}} I_{\text{Public}}$	0.056	55.8%
$\lambda_{\text{Private}} I_{\text{Private}}$	0.019	19.5%
Clermont-Ferrand	**Entropy share**	**% of school segregation**
Total school segregation	0.085	100%
$I_{\text{Public vs. Private}}$	0.017	19.9%
$\lambda_{\text{Public}} I_{\text{Public}}$	0.054	63.5%
$\lambda_{\text{Private}} I_{\text{Private}}$	0.014	16.6%

2.3 The contribution of school circumvention strategies to school segregation

We thus propose a decomposition that allows us to isolate the respective contributions of residential segregation, and of the circumvention of the restrictive zoning, to school segregation. The principle of the decomposition (which is detailed in the Appendix A) relies on different partitions of segregation, first at the school level, and second at the residential level (according to the school zoning). It requires to know both the school where pupils should have been enrolled (according to the restrictive zoning) and the school they actually attend.

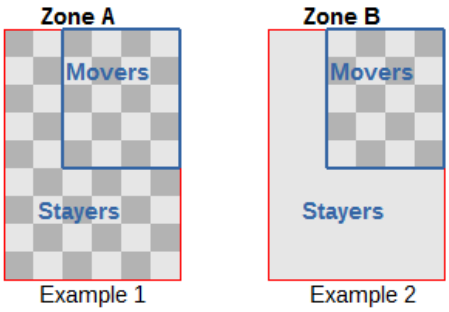
As a first step, we only consider two types of pupils: those who respect the zoning (and thus actually enroll in the public school that corresponds to their home location), indexed hereafter by S (as for stayers), and those who circumvent the school zoning, indexed by M (as for movers) irrespective of whether they attend a private school or a public school different from their assigned one. We may thus link H_{Sch}, the segregation index measured among schools (determined by pupils' actual enrollment) to H_Z, the segregation index measured among residential areas, as defined by the school zoning (determined by pupils' residential location), in the following way:

$$H_{Sch} = H_Z + \Delta H^M \tag{3}$$

where ΔH^M corresponds to the contribution of the circumvention of the school zoning to school segregation. It can be shown that ΔH^M is the sum of three terms (proofs in the Appendix A):

$$\Delta H^M = \underbrace{\sum_z \lambda_z H_z^{S\ vs\ M}}_{(A)} \underbrace{- \sum_{sch} \lambda_{sch} H_{sch}^{S\ vs\ M}}_{(B)} \underbrace{+ \lambda^M (H_{Sch}^M - H_Z^M)}_{(C)} \tag{4}$$

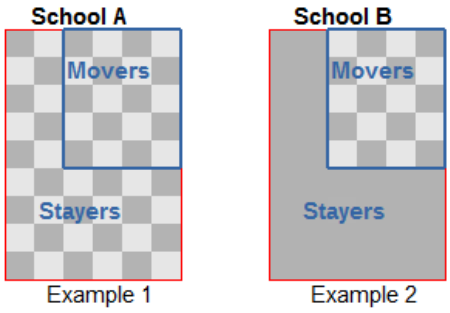
The first term, $(A) = \sum_z \lambda_z H_z^{S\ vs\ M}$, reflects the local social segmentation between stayers



For example, dark grey blocks represent advantaged pupils and light grey one represent disadvantaged pupils.

Figure 2: Example of cream-skimming effect

and movers, within each zone z. $H_z^{S\ vs\ M}$ is the entropy index measured between stayers S and movers M when restricting the sample to zone z (λs denoting weights). The discrepancy between H_{Sch} and H_Z may therefore arise if within each school zone, the pupils who circumvent the local school differ in terms of social background from those who actually attend it. Not only does the social background of pupils matter, but also their residence area: the fact that an advantaged pupil opting out from its assigned school has lower detrimental effect on the school mix in this school if he or she leaves from an advantaged school zone rather than a disadvantaged one. This potential "cream-skimming effect" is depicted on Figure 2: Example 1 depicts zone A, composed equally of advantaged pupils (dark grey blocks) and disadvantaged pupils (light grey blocks). The social composition of movers is the same as that of stayers, so in the case of this zone A we have $\mathcal{H}_A^{S\ vs\ M} = 0$. In Example 2, movers of zone B have the same social composition as movers of zone A, but they leave behind pupils who are much more disadvantaged, thus creating social segregation: for zone B, $\mathcal{H}_B^{S\ vs\ M} > 0$.

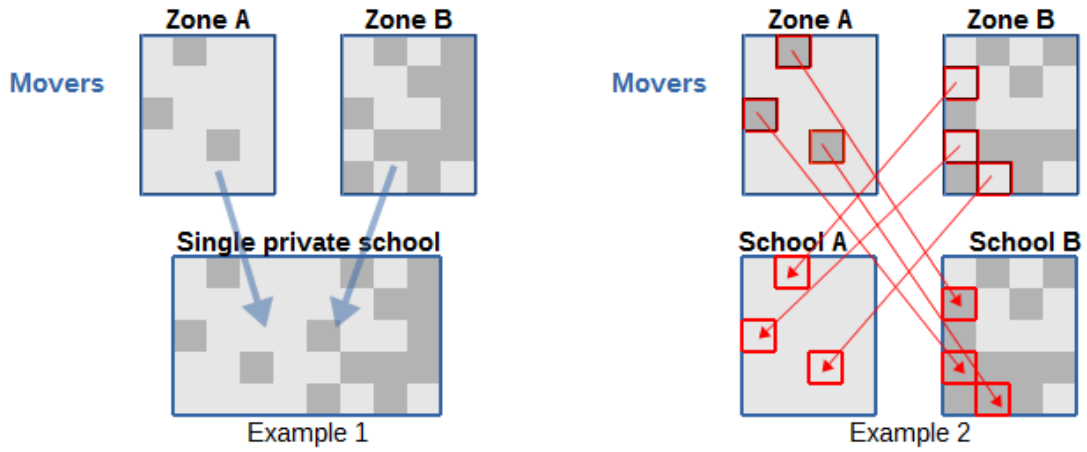


For example, dark grey blocks represent advantaged pupils and light grey ones represent disadvantaged pupils.

Figure 3: Example of diversity effect

Conversely, pupils circumventing their local school may bring social diversity to the school they join, if their social background differs from that of the pupils in the school joined. $(B) = -\sum_{sch} \lambda_{sch} H_{sch}^{S\ vs\ M}$ translates this diversity effect: $H_{sch}^{S\ vs\ M}$ is the entropy index measured between movers and stayers enrolled in school *sch*. For a given school *sch*, it increases when pupils enrolled in *sch* while theoretically assigned to another school (the movers) are socially different from pupils who are enrolled in this school because of the zoning (the stayers). Whereas (A) is mechanically positive, (B) is mechanically negative, and the two terms may offset each other. Terms (A) and (B) illustrate the *a priori* ambiguous contribution of school choice to segregation

in schools, when school choice takes the form of circumventing the assigned local school. On the one hand, it may increase the level of segregation because in a neighborhood, pupils who opt out of the assigned school may differ from the ones who enroll in this school. On the other hand, school choice may decrease the segregation level if it allows children from disadvantaged or middle background to enroll in more privileged schools. This potential diversity effect is illustrated on Figure 3. Here again, movers have the same social composition in examples 1 and 2. Whereas their arrival in school A is neutral in terms of segregation ($H_A^{S\ vs\ M} = 0$), Example 2 presents a diversity effect when movers arrive in very advantaged school B and bring some diversity: we therefore have $-H_B^{S\ vs\ M} < 0$.



For example, dark grey blocks represent advantaged pupils and light grey one represent disadvantaged pupils.

Figure 4: Net social segmentation among movers

Term $(C) = \lambda^M(H_{Sch}^M - H_Z^M)$ translates the idea that pupils circumventing their local school are eventually sorted among schools.H_{Sch}^M is the entropy index measured between movers, when considering their repartition among the schools they eventually attend. It is mitigated by H_Z^M, the entropy index between movers measured among residential zones. For example, (C) may be negative if some pupils from a privileged background and others from a disadvantaged background who decided to circumvent their respective schools assigned by the zoning, eventually enroll in the same school. A social diversity effect of this type appears in Example 1 of Figure 4, which takes the case of a single private school mixing movers from two different zones, who are socially different ($\lambda^{PR}(\mathcal{H}_{Sch}^{PR} - \mathcal{H}_Z^{PR}) < 0$). On the other hand, example 2 shows a case where schools A enrolls the disadvantaged pupils from zone A and some disadvantaged pupils from zone B, and conversely school B enrolls mostly advantaged pupils from zone B, plus some advantaged pupils from zone A (here we focus on movers only). This "social specialization" of schools brings to an additional social stratification compared to the initial level of residential segregation among movers and we would therefore have $\lambda^{PR}(\mathcal{H}_{Sch}^{PR} - \mathcal{H}_Z^{PR}) > 0$.

2.4 Circumvention to private vs. public schools

We can further detail this decomposition, by comparing the component due to circumvention to private schooling, from circumvention to another public school. If we denote those who circumvent the zoning for a private school $PR = M^{PR}$ (movers to private school) and those whose circumvent the zoning for a public school M^{PU} (movers to a public school), and finally $PU = M^{PU} + S$ all pupils attending a public school (whether it is their assigned one or not). It may then be shown that (see the proof in the Appendix A):

$$\mathcal{H}_{Sch} = \mathcal{H}_Z + \Delta\mathcal{H}_{M^{PR}} + \Delta\mathcal{H}_{M^{PU}} \tag{5}$$

with :

$$\Delta\mathcal{H}_{M^{PR}} = \underbrace{\sum_z \lambda_z \mathcal{H}_z^{PR\ vs\ PU}}_{\text{1: cream-skimming effect (=A)}} + \underbrace{\lambda^{PR}(\mathcal{H}_{Sch}^{PR} - \mathcal{H}_Z^{PR})}_{\text{2: reallocation effect (=C)}} \tag{6}$$

$$\Delta\mathcal{H}_{M^{PU}} = \underbrace{\sum_z \lambda_z \mathcal{H}_z^{M^{PU}\ vs\ S}}_{\text{1': cream-skimming effect (=A)}} \underbrace{- \sum_{sch} \lambda_{sch} \mathcal{H}_{sch}^{M^{PU}\ vs\ S} + \lambda^{M^{PU}}(\mathcal{H}_{Sch}^{M^{PU}} - \mathcal{H}_Z^{M^{PU}})}_{\text{2': reallocation effect (=B+C)}} \tag{7}$$

The interpretation is the same as before, with (1) and (2) the counterparts for private schools of terms (A) and (C) of 4; and (1') and (2') the counterparts for movers to public schools of (A) and (B)+(C) respectively.

Two remarks should be made. First, all pupils enrolled in private schools are by definition movers in our taxonomy (private schools do not belong to the state school zoning), therefore movers do not mix with stayers when they enroll in private schools. This explains why for private schools, we do not have a counterpart of the second term (B) in the general decomposition 4: the only existing reallocation effect takes place *within* movers to private schools. For movers to public schools, we designate as "reallocation" effect the combination of the diversity effect (B) in the schools joined and of the "social specialization" effect (C) among movers to public schools.

Second, the first term is not computed in exactly the same way for circumvention to private and to alternative public schools in the decomposition proposed in 7. In this decomposition, we first consider pupils enrolled in private school separately from all others,[9] and then compare the impact of circumvention from assigned school *restricted to pupils enrolled in public school*. Thus, for the same residential zone z, for $\Delta H_{M^{PR}}$ we single out pupils enrolled in a private school among all pupils living in this zone, while in $\Delta H_{M^{PU}}$ we single out on pupils enrolled in an alternative school on resident pupils, once excluded the pupils enrolled in private school. We can choose an alternative order and first single out pupils who circumvent to alternative school, and then compare stayers with movers to a private school. The computation has no reason to be identical, but we check that our conclusions are not driven by this.

3 Results

The detailed decomposition is performed in each of the three urban areas considered: Paris, Bordeaux and Clermont-Ferrand (Tables 4, 5 and 6). In all cases, the circumvention of zoning contributes to increase social segregation measured at the school level, when compared to the one than prevails at the residential level, by more than half: +59.1% for Clermont-Ferrand, +64.5% for Bordeaux and as much as +96.0% for Paris where segregation measured at the school zoning level is around half only as that measured among schools. The magnitude of the effects between the three urban areas cannot be directly compared, given the different perimeters considered (strict city boundaries for Paris, wider urban areas for Bordeaux and Clermont-Ferrand) and the population densities implied.

In all three districts, the contribution of circumvention strategies to school segregation is mostly due to pupils circumventing to private schools. Within this contribution of private schools, the most important contribution is term (1) in 6, which is the cream skimming-effect.

[9] It should be noted that the global contribution of the private sector $\Delta\mathcal{H}_{M^{PR}}$ can be computed exactly with simulation techniques, as mentioned before. The same holds for the contribution of school choice in public sector.

	Entropy	Std. Error	% of (Z)
Residential segregation $(\mathcal{H}_Z)$	0.076	(0.0024)	
School segregation $(\mathcal{H}_{Sch})$	0.149	(0.0031)	
Contribution of all movers $(\mathcal{H}_{Sch} - \mathcal{H}_Z = \Delta\mathcal{H}_{M^{PR}} + \Delta\mathcal{H}_{M^{PU}})$	0.073 (***)	(0.0025)	96.0%
Movers to private schools $(\Delta\mathcal{H}_{M^{PR}} = (1) + (2))$	0.066 (***)	(0.0023)	87.2%
M^{PR} cream-skimming effect (1)	0.059	(0.0020)	77.9%
M^{PR} reallocation effect (2)	0.007 (*)	(0.0015)	9.2%
Movers to public schools $(\Delta\mathcal{H}_{M^{PU}} = (1') + (2'))$	0.007 (***)	(0.0014)	8.9%
M^{PU} cream-skimming effect $(1')$	0.012	(0.0011)	15.4%
M^{PU} reallocation effect $(2')$	-0.005 (***)	(0.0013)	-6.6%

Note : (.) p<0.1; (*) p<0.05; (**)p<0.01; (***)p<0.001; (-) NS
No statistical significance indication is given for terms that are by definition positive or negative.
Standard errors are computed by bootstrap.

Table 4: Paris

Within each school zone, the pupils who circumvent their local school to attend a private one have a social composition that is very different from that of pupils remaining in the public sector. Thus, the set of pupils they "leave behind" is more homogeneous in terms of social composition than it was initially in the zone. This local public-private social segmentation accounts for +77.9 percentage points (out of +96.0%) in Paris, +51.3% in Bordeaux and +52.3% in Clermont-Ferrand.

	Entropy	Std. Error	% of (Z)
Residential segregation $(\mathcal{H}_Z)$	0.060	(0.0033)	
School segregation $(\mathcal{H}_{Sch})$	0.099	(0.0040)	
Contribution of all movers $(\mathcal{H}_{Sch} - \mathcal{H}_Z = \Delta\mathcal{H}_{M^{PR}} + \Delta\mathcal{H}_{M^{PU}})$	0.039 (***)	(0.0031)	64.5%
Movers to private schools $(\Delta\mathcal{H}_{M^{PR}} = (1) + (2))$	0.033 (***)	(0.0027)	54.8%
M^{PR} cream-skimming effect (1)	0.031	(0.0025)	51.3%
M^{PR} reallocation effect (2)	0.002 (-)	(0.0019)	3.6%
Movers to public schools $(\Delta\mathcal{H}_{M^{PU}} = (1') + (2'))$	0.006 (***)	(0.0017)	9.7%
M^{PU} cream-skimming effect $(1')$	0.012	(0.0016)	19.6%
M^{PU} reallocation effect $(2')$	-0.006 (***)	(0.0018)	-9.9%

Note : (.) p<0.1; (*) p<0.05; (**)p<0.01; (***)p<0.001; (-) NS
No statistical significance indication is given for terms that are by definition positive or negative.
Standard errors are computed by bootstrap.

Table 5: Bordeaux

In the tree cities considered here, these locally segregative effects are not mitigated by at the aggregated level. The reallocation effect, ie. term (2) of the decomposition 6 may be negative in principle if pupils who are enrolled in private education are less stratified socially according to the school they attend than according to their residential neighborhood (which would correspond to a social diversity effect among pupils enrolled in private schools). According to our data, in Bordeaux and Clermont-Ferrand, this contribution is very small and not significantly different from 0 at the 10% level. For those urban areas, social segmentation between private schools is mostly explained by preexisting residential segregation among the pupils who circumvent to those private schools. In Paris there is even a slight additional stratifying effect of private education (+9.2%, significant at the 5% level). Far from allowing diversity *among private school pupils*, the private sector contributes to sort them even more between middle schools. The difference observed between districts may be due to various local policies of private education.

The circumvention to another public school (than the one assigned by the school zoning)

	Entropy	Std. Error	% of (Z)
Residential segregation $(\mathcal{H}_Z)$	0.053	(0.0050)	
School segregation $(\mathcal{H}_{Sch})$	0.085	(0.0060)	
Contribution of all movers $(\mathcal{H}_{Sch} - \mathcal{H}_Z = \Delta\mathcal{H}_{M^{PR}} + \Delta\mathcal{H}_{M^{PU}})$	0.031 (***)	(0.0054)	59.1%
Movers to private schools $(\Delta\mathcal{H}_{M^{PR}} = (1) + (2))$	0.028 (***)	(0.0047)	52.9%
M^{PR} cream-skimming effect (1)	0.028	(0.0039)	52.3%
M^{PR} reallocation effect (2)	0.000 (-)	(0.0032)	0.6%
Movers to public schools $(\Delta\mathcal{H}_{M^{PU}} = (1') + (2'))$	0.003 (-)	(0.0032)	6.2%
M^{PU} cream-skimming effect $(1')$	0.014	(0.0027)	26.8%
M^{PU} reallocation effect $(2')$	-0.011 (***)	(0.0032)	-20.7%

Note : (.) p<0.1; (*) p<0.05; (**)p<0.01; (***)p<0.001; (-) NS
No statistical significance indication is given for terms that are by definition positive or negative.
Standard errors are computed by bootstrap.

Table 6: Clermont-Ferrand

contributes only slightly to increase segregation : the overall contribution is +8.9% for Paris, +9.7% for Bordeaux and +6.2% for Clermont-Ferrand (not significantly different from 0 at the 10% level in the latter case). In each case, the cream-skimming effect implied by movers to public schools (term (1'): +15.4% and +19.6% respectively for Paris and Bordeaux) is not entirely compensated by the reallocation effect.

4 Conclusion

In this paper, we aim to determine the respective contributions of residential segregation (as defined by restrictive school zoning) and selective circumvention strategies, to the level of social segregation among middle school pupils in three French urban areas (Paris, Bordeaux, Clermont-Ferrand) in 2015. We show that pupils circumventing their local public school in order to attend a private school contribute to increase social segregation by 50% at least with respect to segregation measures at the residential level. Local social segmentation between public and private school students is the main driver of this additional segregation. Furthermore, private schooling does not allow more social diversity, even *among* those who circumvent to private schools. Although the social composition of movers to another public school is close to the average, this type of strategies contribute to slightly increase segregation, by 5%-10% on the three French urban areas considered here.

References

Rebecca Allen. Allocating Pupils to Their Nearest Secondary School: The Consequences for Social and Ability Stratification. *Urban Studies*, 44(4):751–770, April 2007.

Aude Cadoret. La mixité sociale des collèges de Clermont-Ferrand. *Géographie de l'école*, pages 96–105, 2017.

Gabrielle Fack and Julien Grenet. Rapport d'évaluation de l'assouplissement de la carte scolaire. 2012.

Gabrielle Fack, Julien Grenet, and Asma Benhenda. L'impact des procédures de sectorisation et d'affectation sur la mixité sociale et scolaire dans les lycées d'Île-de-France. *Rapport IPP*, 2014.

David M. Frankel and Oscar Volij. Measuring school segregation. *Journal of Economic Theory*, 146(1):1–38, 2011.

Jean-Christophe François. Évitement à l'entrée en sixième et division de l'espace scolaire à Paris. *L'espace géographique*, 31:307–327, 2002.

Harry B.G. Ganzeboom, Paul M. De Graaf, and Donald J. Treiman. A standard international socio-economic index of occupational status. *Social Science Research*, 21(1):1–56, 1992.

Pauline Givord, Marine Guillerm, Olivier Monso, and Fabrice Murat. La ségrégation sociale entre les collèges : quelles différences entre public et privé, aux niveaux national, académique et local ? *Éducation et formations*, (91):53–76, 2016.

R. Harris, R. Johnston, S. Burgess, and D. Wilson. Modelling school catchments for segregation studies, 2007.

Chang-Tai Hsieh and Miguel Urquiola. The effects of generalized school choice on achievement and stratification: Evidence from Chile's voucher program. *Journal of public Economics*, 90 (8):1477–1503, 2006.

David R. James and Karl E. Taeuber. Measures of segregation. *Sociological Methodology*, 15: 1–32, 1985.

Ron Johnston, Simon Burgess, Deborah Wilson, and Richard Harris. School and Residential Ethnic Segregation: An Analysis of Variations across England's Local Education Authorities. *Regional Studies*, 40(9):973–990, 2006.

Douglas S. Massey and Nancy A. Denton. The dimensions of residential segregation. *Social Forces*, 67(2):281–315, 1988.

Pierre Merle. La carte scolaire et son assouplissement. Politique de mixité sociale ou de ghettoïsation des établissements ? *Sociologie*, 2(1):37–50, 2011.

Pauline Musset. School Choice and Equity, Current Policies in OECD Countries and a Literature Review. *OECD Education Working Papers*, 66, 2012.

OECD. The learning environment and organisation of schools – School Choice. *Education at a Glance – OECD Indicators*, 2010.

Ann Owens, Senn Reardon, and Christopher Jencks. Income Segregation between Schools and School Districts. *CEPA Working Paper*, (16-04), 2016.

Beatrice Schindler Rangvid. Living and learning separately? Ethnic segregation of school children in Copenhagen. *Urban Studies*, 44(7):1329–1354, 2007.

Sean F. Reardon and Glenn Firebaugh. Measures of multigroup segregation. *Sociological Methodology*, 32(1), 2002.

Andrea Riedel, Kerstin Schneider, Claudia Schuchart, and Horst Weishaupt. School choice in German primary schools: How binding are school districts? *Journal for Educational Research Online / Journal für Bildungsforschung Online*, 2(1):94–120, 2010.

Thierry Rocher. Construction d'un indice de position sociale des élèves. *Éducation et formations*, 90:5–27, 2016.

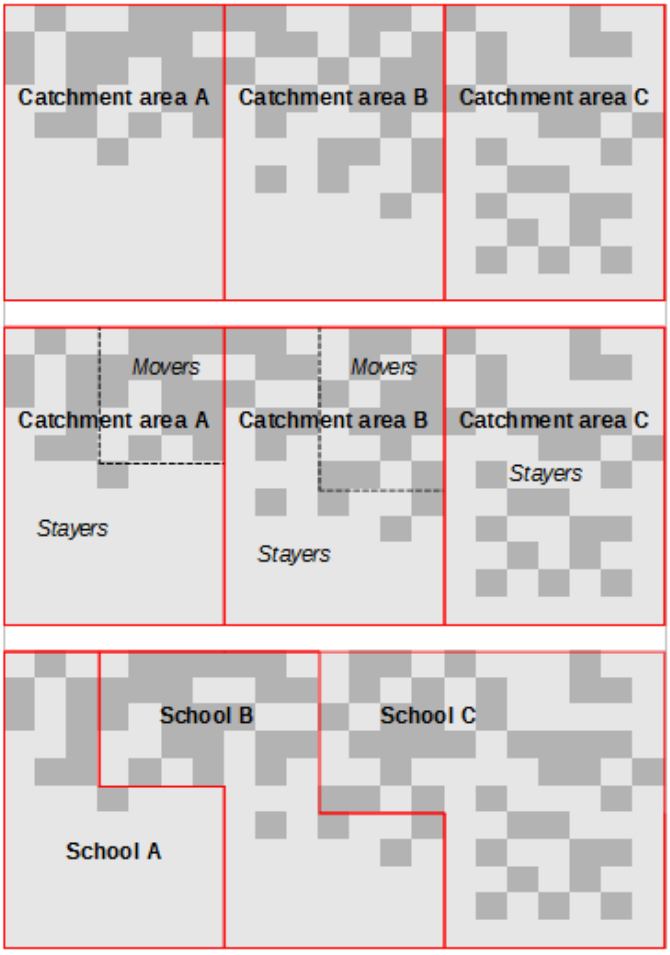
Martin Söderström and Roope Uusitalo. School Choice and Segregation: Evidence from an Admission Reform. *Scandinavian Journal of Economics*, 112(1):55–76, 2010.

Michèle Thaurel-Richard and Fabrice Murat. Évolution des caractéristiques des collèges durant l'assouplissement de la carte scolaire de 2007. *Éducation et formations*, (83):11–23, 2013.

A Decomposition

For a set of R zones, we have Rx3 initial groups defined by a school zone and a status (circumventing to a state school /circumventing to a private school / enrollment in the assigned school). The principle of the segregation relies on the fact that we can reorder these partitions of R*3 groups in two ways: the one depending on school zones, the other depending on actual school. We can thus define a segregation index measured when using the full partition, and decomposed it in either residential units, either school units. Figure 5 illustrates in a case with only avoiders to the public: pupils who circumvent in the A zone enroll in the B school and in the zone B those who circumvent enroll in the C school. The measure of segregation corresponding to these full partition is not an object of interest in itself, but is used to compare the measures of residential segregation and school segregation.

Figure 5: Illustration : from residential segregation to school segregation



Each tile corresponds to a pupil, for example the favored ones in darker. The five subgroups (middle) can be divided into three sectors (above) or three colleges (bottom). The total segregation between the five subgroups can thus be linked, on the one hand, to segregation between sectors and, on the other hand, to segregation between colleges.

Let us denote by $\mathcal{H}_Z^T$ the entropy index calculated for the full partition of all entities corresponding of the fact of crossing both type k (stayers S, movers to state school M^{PU}, movers to private school M^{PR}) and residential zones z. This index that uses the two dimensions is used only for the sake of calculation (as it does not have direct interpretation in itself).

$$\mathcal{H}_Z^T = \sum_z \sum_{k\in(S,\ M^{PU},\ M^{PR})} \pi_z^k \frac{h(P) - h(P_z^k)}{h(P)}$$

where P is the distribution of pupils in the different socio-economic groups in the whole population, P_z^k are the corresponding distributions in the zone z for pupils of type k, and π_z^k the proportions of these pupils in the whole population. Specifically, we have $\pi_z^k = \frac{\mathbf{N}[(k,z)]}{N_T}$ where $\mathbf{N}[(k,z)] = Card(i \text{ of type } k \text{ and in zone } z)$ and N_T the size of the total population. We can decompose this index $\mathcal{H}_Z^T$ in two different ways, first by focusing on zones:

$$\begin{aligned}
\mathcal{H}_Z^T =& \underbrace{\sum_z \pi_z \frac{h(P) - h(P_z)}{h(P)}}_{\mathcal{H}_Z} + \sum_z \frac{h(P_z)}{h(P)} \sum_{k\in(S,\ M^{PU},\ M^{PR})} \pi_z^k \frac{h(P_z) - h(P_z^k)}{h(P_z)} \\
=& \mathcal{H}_Z + \sum_z \frac{h(P_z)}{h(P)} \underbrace{\frac{\mathbf{N}[(M^{PR}, z)]}{N_T} \frac{h(P_z) - h(P_z^{M^{PR}})}{h(P_z)} + \frac{\mathbf{N}[(M^{PU} \text{ or } S, z)]}{N_T} \frac{h(P_z) - h(P_z^{M^{Pu}+S})}{h(P_z)}}_{\mathcal{H}_z^{M^{Pr}vs(M^{Pu}+S)}} \\
&+ \sum_z \frac{\mathbf{N}[(M^{PU} \text{ or } S, z)]}{N_T} \frac{h(P_z^{M^{Pu}+S})}{h(P)} \underbrace{\frac{\mathbf{N}[(M^{PU}, z)]}{\mathbf{N}[(M^{PU} \text{ or } S, z)]} \frac{h(P_z^{M^{Pu}+S}) - h(P_z^{M^{Pu}})}{h(P_z^{M^{Pu}+S})} + \frac{\mathbf{N}[(S, z)]}{\mathbf{N}[(M^{PU} \text{ or } S, z)]} \frac{h(P_z^{M^{Pu}+S}) - h(P_z^S)}{h(P_z^{M^{Pu}+S})}}_{\mathcal{H}_{z,(M^{Pu}+S)}^{M^{Pu}vsS}} \\
=& \mathcal{H}_Z + \sum_z \left[\lambda_z^1 \mathcal{H}_z^{M^{Pr}vs(M^{Pu}+S)} + \lambda_z^2 \mathcal{H}_{z,(M^{Pu}+S)}^{M^{Pu}vsS} \right]
\end{aligned}$$

with $\mathcal{H}_Z$ the entropy index corresponding to residential segregation (considering all pupils of one residential areas), $\mathcal{H}_z^{M^{Pr}vs(M^{Pu}+S)}$ the entropy index restricting the sample to pupils living in residential area z, and comparing the distribution of social groups among pupils enrolled in private school with the one among pupils enrolled in public school, $\mathcal{H}_{z,(M^{Pu}+S)}^{M^{Pu}vsS}$ the entropy index restricting the sample to pupils living in residential area z *and* enrolled in public school, and comparing the distribution of social groups among pupils enrolled in the assigned public school with the one among pupils circumventing toward another public school. The terms λ_z^1 and λ_z^2 correspond to weights.[10]

By now decomposing this very same index $\mathcal{H}_Z^T$ by types k (instead of zones) we have also:

$$\begin{aligned}
\mathcal{H}_Z^T =& \underbrace{\sum_k \pi^k \frac{h(P) - h(P^k)}{h(P)}}_{\mathcal{H}^K} + \sum_k \frac{h(P^k)}{h(P)} \pi^k \underbrace{\sum_z \pi_{k,z}^k \frac{h(P^k) - h(P^k z)}{h(P^k)}}_{\mathcal{H}_Z^k} \\
=& \mathcal{H}^K + \sum_k \lambda^k \mathcal{H}_Z^k
\end{aligned}$$

with $\mathcal{H}^K$ denotes the entropy index considering a partition depending on the type (stayers, movers to public, movers to private school) only, and $\mathcal{H}_z^k$ the entropy index measuring residential segregation, when restricting the sample to one type k.

We can also do the same type of decompositions by considering the full partition of all entities corresponding now the crossing of both type k and school s.

When considering now entities corresponding to types and enrollment school (instead of

[10] We have $\lambda_z^1 = \frac{h(P_z)}{h(P)}$ and $\lambda_z^2 = \frac{h(P_z^{M^{PU}+S})}{h(P)} \pi_z^{M^{PU}+S}$

residential zoning), we have a similar decomposition:

$$\begin{aligned}\mathcal{H}^T_{Sch} =& \sum_s \sum_k \pi^k_s \frac{h(P) - h(P^k_s)}{h(P)} \\ =& \underbrace{\sum_s \pi_s \frac{h(P) - h(P_s)}{h(P)}}_{\mathcal{H}_{Sch}} + \sum_s \frac{h(P_s)}{h(P)} \sum_{k \in (S,\, M^{PU},\, M^{PR})} \pi^k_s \frac{h(P_s) - h(P^k_s)}{h(P_s)}\end{aligned}$$

It is useful to remark some simplifications. In private schools, all students are of type $k = M^{PR}$ and in public ones, they are either of type S or M^{PU}. Compared to the decomposition above, the second term of this decomposition is thus null. Using similar notation as above we have thus

$$\begin{aligned}\mathcal{H}^T_{Sch} =& \mathcal{H}_{Sch} + \sum_s \lambda^2_s \mathcal{H}^{M^{Pu} vs S}_{s,(M^{Pu}+S)} \\ =& \mathcal{H}^K + \sum_k \lambda_k \mathcal{H}^K_{Sch}\end{aligned}$$

the second line providing when considering only the partition by type of students. We thus have:

$$\begin{aligned}\mathcal{H}^K =& \mathcal{H}_Z + \sum_z \left[\lambda^1_z \mathcal{H}^{M^{Pr} vs (M^{Pu}+S)}_z + \lambda^2_z \mathcal{H}^{M^{Pu} vs S}_{z,(M^{Pu}+S)}\right] - \sum_k \lambda^k \mathcal{H}^K_Z \\ =& \mathcal{H}_{Sch} + \sum_s \lambda^2_s \mathcal{H}^{M^{Pu} vs S}_{s,(M^{Pu}+S)} - \sum_k \lambda^k \mathcal{H}^K_{Sch}\end{aligned}$$

Finally, we obtain:

$$\mathcal{H}_{Sch} = \mathcal{H}_Z + \underbrace{\sum_z \lambda^1_z H^z_{M^{Pr} vs (M^{Pu}+S)} + \lambda^{PR}[\mathcal{H}^{M^{Pr}}_{Sch} - \mathcal{H}^{M^{Pr}}_Z]}_{\Delta \mathcal{H}_{M^{Pr}}} + \underbrace{\sum_z \lambda^2_z \mathcal{H}^{M^{Pu} vs S}_{z,(M^{Pu}+S)} - \sum_s \lambda^2_s \mathcal{H}^{s,(M^{Pu}+S)}_{M^{Pu} vs S} + \lambda^{PU}[\mathcal{H}^{M^{Pu}}_{Sch} - \mathcal{H}^{M^{Pu}}_Z]}_{\Delta \mathcal{H}_{M^{Pu}}}$$

as by definition of the stayers (students enrolled in the school assigned by the zoning), $\mathcal{H}^S_{Sch} = \mathcal{H}^S_Z$ and denoting $\lambda^{PU} = \lambda^{M^{PU}}$ and $\lambda^{PR} = \lambda^{M^{PR}}$.

B Robustness analysis: Social position index and variance ratio

The database also provides a continuous index of social position of the household, the social position index (SPI) introduced by Rocher [2016]. The construction of this social position shares some similarities with the one of the International Socio-Economic Index of occupational status (ISEI) proposed by Ganzeboom et al. [1992], but focus on the determinant of schooling achievement of children, whereas Ganzeboom et al. [1992] focus on the correlation between occupation and income, for a given level of educational attainment. The social position index assigns scores to occupation categories from the French occupational categories (*professions et catégories socioprofessionelles*, PCS) as a main determinant of children attainment.[11] This

[11] In practice, the scaling was perform using a detailed database that contains a large set of variables related to parental education, material conditions, cultural capital, ambition and parental involvement in scholarship of their children, as well as the score of children to standardized tests. These variables are synthesized using multiple correspondence analysis, and a social position score is then attributed to each occupation as described in common classification depending on its position in the first axis of this classification.

social position index has the advantage of providing a scaling of all occupations, depending on the observed proximity with schooling achievement of children. It is highly correlated for instance with the ISEI, but differ for some occupation : for instance, it provides a higher score to teachers and professionals, and a smaller one to managers, than the ISEI. This is consistent with the previous results in sociological literature for France : teachers earn income close to the average, but their children usually perform amongst the best at school.

Compared to the common classification of occupations, the SPI reduces socioeconomic status to a single dimension, but provides a better scaling of social distances between groups. Rocher [2016] also proposes a version of the SPI that allows to combine the occupation of both parents. This is key advantage : mother occupation may be a good proxy for maternal education, usually a key determinant in children schooling achievement.

As an alternative to our four-position socioeconomic status, we also use this continuous social position index. When dealing with a continuous social position index, a natural measure of segregation is the variance ratio index (see James and Taeuber [1985] for a binary version[12]). Here again, segregation corresponds to the association between schools and socioeconomic background. For instance, considering a partition in K groups ($k = 1 \dots K$) of N_T pupils ($i = 1 \dots N_T$), the variance index (between component) corresponds to:

$$\mathcal{V} = \frac{\sum_{k=1}^{K} \pi^k (\bar{y}^k - \bar{y})^2}{\frac{1}{N_T} \sum_{i=1}^{N_T} (y_i - \bar{y})^2}$$

where $\bar{y}$ is the average position social index in the whole sample, $\bar{y}^k$ this average measured in the group k and $\pi^k = \frac{n_k}{N_T}$ the share of pupils from group k.

The decomposition properties of this measure are the same as for the entropy index, and we perform the same type of decomposition. The results obtained are similar.

Let decompose the total variance observed in the whole sample depending on the residential district:

$$\begin{aligned}
\frac{1}{N_T}\sum_i (y_i - \bar{y})^2 &= \frac{1}{N_T}\sum_z \sum_{i\in z} (y_i - \bar{y}_z)^2 + \frac{1}{N_T}\sum_z n_z (\bar{y}_z - \bar{y})^2 \\
&= \frac{1}{N_T}\sum_z \sum_{k\in(M^{Pr},M^{Pu},S)} \sum_{i\in z\&k} (y_i - \bar{y}_z^k)^2 + \frac{1}{N_T}\sum_z \sum_{k\in(M^{Pr},M^{Pu},S)} n_z^k (\bar{y}_z^k - \bar{y}_z)^2 + \mathcal{V}_Z \frac{1}{N_T}\sum_i (y_i - \bar{y})^2 \\
&= \frac{1}{N_T} \sum_{k\in(M^{Pr},M^{Pu},S)} \sum_z \sum_{i\in z\&k} (y_i - \bar{y}_z^k)^2 + \frac{1}{N_T}\sum_z \left[n_z^{M^{Pr}} (\bar{y}_z^{M^{Pr}} - \bar{y}_z)^2 + n_z^{M^{Pu}+S} (\bar{y}_z^{M^{Pu}+S} - \bar{y}_z)^2 \right] \\
&\quad + \frac{1}{N_T}\sum_z \left[n_z^{M^{Pu}} (\bar{y}_z^{M^{Pu}} - \bar{y}_z^{M^{Pu}+S})^2 + n_z^S (\bar{y}_z^S - \bar{y}_z^{M^{Pu}+S})^2 \right] + \mathcal{V}_Z \frac{1}{N_T}\sum_i (y_i - \bar{y})^2 \\
&= \frac{1}{N_T} \sum_{k\in(M^{Pr},M^{Pu},S)} \mathcal{V}_{inter.z}^k \sum_{i\in k} (y_i - \bar{y}^k)^2 + \frac{1}{N_T}\sum_z \mathcal{V}_z^{M^{Pr} vs (M^{Pu}+S)} \sum_{i\in z} (y_i - \bar{y}_z)^2 \\
&\quad + \frac{1}{N_T}\sum_z \mathcal{V}_z^{M^{Pu} vs S} \sum_{i\in z\&(M^{Pu}+S)} (y_i - \bar{y}_z^{M^{Pu}+S})^2 + \mathcal{V}_Z \frac{1}{N_T}\sum_i (y_i - \bar{y})^2
\end{aligned}$$

where $\bar{y}$ is the social index averaged over the whole sample, $\bar{y}_z$ over the pupils in the residential district z, and $\bar{y}_z^k$ over those in residential district of type k (meaning stayers, movers to a private school or mover to another state school), and n_z and n_z^k are the corresponding numbers of students, with $\mathcal{V}_Z$ the variance index measuring segregation due to residential districts:

$$\mathcal{V}_Z = \frac{\sum_z \pi_z (\bar{y}_z - \bar{y})^2}{\frac{1}{N_T}\sum_i (y_i - \bar{y})^2}$$

[12]It is well-known that when relying on a binary classification of groups, variance ratio index corresponds to the normalized exposure index.

$\mathcal{V}_z^{M^{Pr}vs(M^{Pu}+S)}$ is the variance ratio inside the residential district r explained by the dichotomy "enrolled in private school" vs "not enrolled in private school", and $\mathcal{V}_z^{SvsM^{Pu}}$ the variance ratio inside the residential district z, restricted to pupils enrolled in public schools, explained by the dichotomy "staying in the assigned public school" vs "circumventing to another public school", and $\mathcal{V}_{inter.z}^k = \frac{\sum_z \sum_{i\in z\&k}(y_i-\bar{y}_z^k)^2}{\sum_{i\in k}(y_i-\bar{y}^k)^2}$ is the variance ratio (within) corresponding to residential district z, when restricting the sample to type k.

We can also perform a similar decomposition, but considering now the partition created by the school where pupils are actually enrolled.

$$\begin{aligned}
\frac{1}{N_T}\sum_i (y_i-\bar{y})^2 &= \frac{1}{N_T}\sum_s \sum_{k\in(M^{Pr},M^{Pu},S)} \sum_{i\in s\&k} (y_i-\bar{y}_s^k)^2 \\
&\quad + \frac{1}{N_T}\sum_s \sum_{k\in(M^{Pr},M^{Pu},S)} n_s^k(\bar{y}_s^k-\bar{y}_s)^2 + \frac{1}{N_T}\sum_s n_s(\bar{y}_s-\bar{y})^2 \\
&= \frac{1}{N_T}\sum_{k\in(M^{Pr},M^{Pu},S)} \sum_s \sum_{i\in s\&k} (y_i-\bar{y}_s^k)^2 + \frac{1}{N_T}\sum_{s \text{ is private}} n_s^{M^{Pr}} \underbrace{(\bar{y}_s^{M^{Pr}}-\bar{y}_s)^2}_{=0} \\
&\quad + \frac{1}{N_T}\sum_{s \text{ is public}} n_s^{M^{Pu}}(\bar{y}_s^{M^{Pu}}-\bar{y}_s)^2 + n_s^S(\bar{y}_s^S-\bar{y}_s)^2 + \mathcal{V}_{Sch}\frac{1}{N_T}\sum_i (y_i-\bar{y})^2 \\
&= \frac{1}{N_T}\sum_{k\in(M^{Pr},M^{Pu},S)} \mathcal{V}_{inter.s}^k \sum_{i\in k}(y_i-\bar{y}^k)^2 \\
&\quad + \frac{1}{N_T}\sum_{s \text{ is public}} \mathcal{V}_s^{M^{Pu}vsS} \sum_{i\in s \text{ is public}} (y_i-\bar{y}_s)^2 + \mathcal{V}_{Sch}\frac{1}{N_T}\sum_i (y_i-\bar{y})^2
\end{aligned}$$

with similar notations as above. It is slightly simpler than in the case above as in private schools, all pupils are by definition of type "movers to private schools" and thus $\bar{y}_s = \bar{y}_s^{M^{Pr}}$, while in the public schools all students are either movers to public or stayers and thus $\bar{y}_s = \bar{y}_s^{M^{Pu+S}}$.

Using both decompositions above, we have:

$$\begin{aligned}
\mathcal{V}_{Sch} = \mathcal{V}_Z &+ \sum_z \lambda_z^{PR}\mathcal{V}_z^{M^{Pr}vs(M^{Pu}+S)} \\
&+ \sum_z \lambda_z^{PU}\mathcal{V}_z^{M^{Pu}vsS} - \sum_s \lambda_s^{PU}\mathcal{V}_z^{M^{Pu}vsS} \\
&+ \sum_{k\in(M^{Pr},M^{Pu},S)} \lambda^k\left[\mathcal{V}_{inter.z}^k - \mathcal{V}_{inter.s}^k\right]
\end{aligned}$$

With $\lambda^k = \frac{\sum_{i\in k}(y_i-\bar{y}^k)^2}{\sum_i(y_i-\bar{y})^2}$, $\lambda_z^{PR} = \frac{\sum_{i\in z}(y_i-\bar{y}_z)^2}{\sum_i(y_i-\bar{y})^2}$, $\lambda_z^{PU} = \frac{\sum_{i\in z\&(M^{Pu}+S)}(y_i-\bar{y}_z^{M^{PU}+S})^2}{\sum_i(y_i-\bar{y})^2}$, $\lambda_s^{PU} = \frac{\sum_{i\in s \text{ is public}}(y_i-\bar{y}_s)^2}{\sum_i(y_i-\bar{y})^2}$

By definition, pupils of type S stay in the state school assigned to their residential district, and thus $\mathcal{V}_{inter.z}^S = \mathcal{V}_{inter.s}^S$. Finally, we obtain (denoting as above $\lambda^{PU} = \lambda^{M^{PU}}$ and $\lambda^{PR} = \lambda^{M^{PR}}$):

$$\begin{aligned}
\mathcal{V}_{Sch} = \mathcal{V}_z &+ \sum_z \lambda_z^{PR}\mathcal{V}_z^{M^{Pr}vs(M^{Pu}+S)} + \lambda^{PR}[\mathcal{V}_{inter.z}^{PR} - \mathcal{V}_{inter.s}^{PR}] \\
&+ \sum_z \lambda_z^{PU}\mathcal{V}_z^{M^{Pu}vsS} - \sum_s \lambda_s^{PU}\mathcal{V}_z^{M^{Pu}vsS} + \lambda^{PU}[\mathcal{V}_{inter.z}^{PU} - \mathcal{V}_{inter.s}^{PU}]
\end{aligned}$$

which similar interpretation as the one relying on the normalized entropy index.